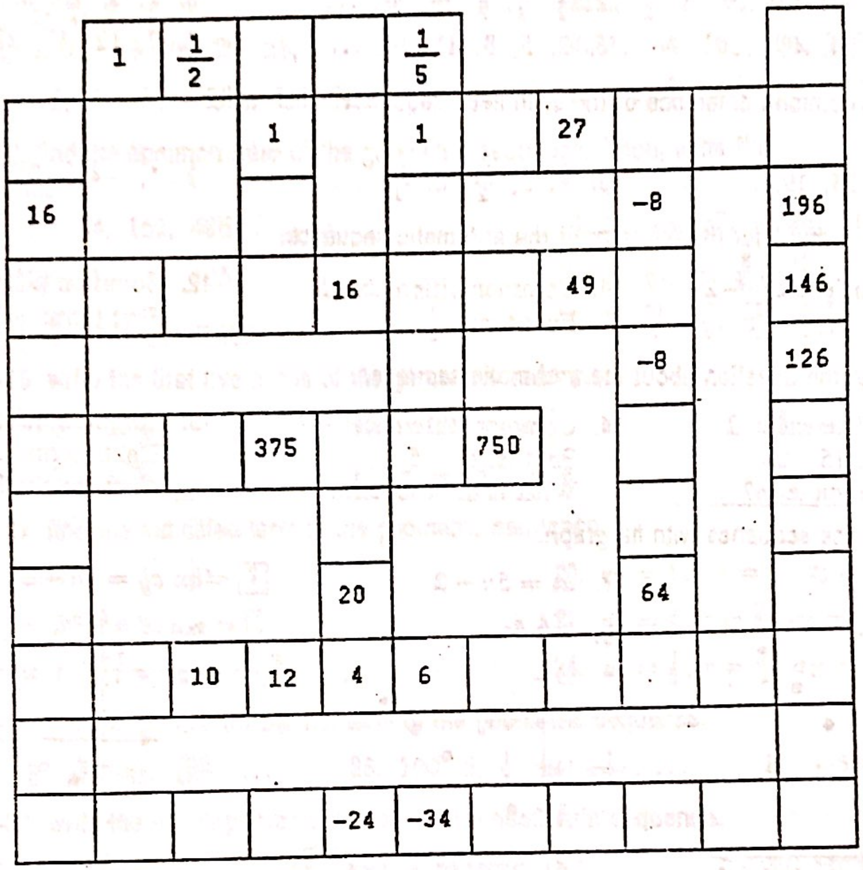
Sequences Puzzle

from the January 1989 issue of *The Mathemtics Teacher*

Going across and going down are different number sequences.
Fill in the empty boxes so that each sequence has a pattern.



Extra Practice 12.2

Name ____________________

In 1–6, decide whether the sequence is arithmetic.

1. 4, 7, 10, 13, 16, ...
2. $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \frac{81}{2}, \ldots$
3. 2, 9, 16, 23, 30, 37, ...
4. 3, 6, 12, 24, 48, ...
5. 2, 5, 8, 11, 14, ...
6. 7, 12, 17, 22, 27, ...

In 7–9, find the common difference of the arithmetic sequence. Then write the next term.

7. 3, 7, 11, 15, 19, ...
8. $\frac{9}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}, \ldots$
9. 1, −4, −9, −14, −19, ...

In 10–12, find a formula for the *n*th term of the arithmetic sequence.

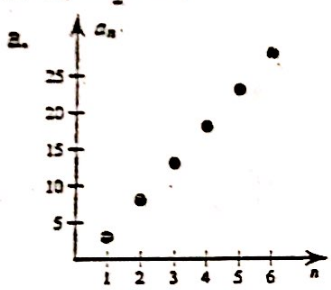
10. Common difference: −2
    First term: 2
11. Common difference: 5
    First term: $\frac{1}{2}$
12. Common difference: 4
    First term: −3

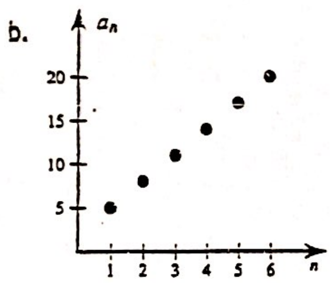
In 13–15, answer the question about the arithmetic sequence.

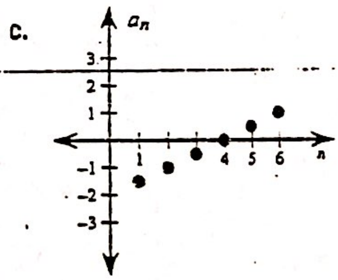
13. Common difference: 2
    Sixth term: 15
    What is the 9th term?
14. Common difference: $-\frac{1}{2}$
    First term: −5
    What is the 10th term?
15. Common difference: 4
    Eighth term: 36
    What are the 1st 4 terms?

In 16–18, match the sequence with its graph.

16. $a_n = \frac{1}{2}n - 2$
17. $a_n = 5n - 2$
18. $a_n = 3n + 2$

a. 

b. 

c. 

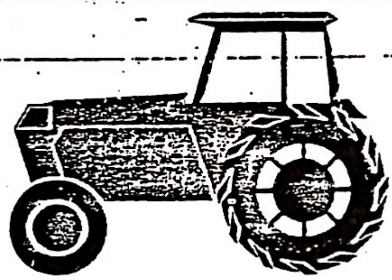
In 19–21, find the sum of the first *n* terms of the arithmetic sequence. Begin with $n = 1$.

19. $a_n = 15 - 3n;\ n = 10$
20. $a_n = -5 + 7n;\ n = 25$
21. $a_n = -\frac{7}{2} + 4n;\ n = 20$

In 22–24, evaluate the sum.

22. $\sum_{i=1}^{25}(4i + 1)$
23. $\sum_{i=1}^{40}(5i - 1)$
24. $\sum_{i=1}^{50}(2i + 5)$

25. *Baling Hay* As a farmer bales a field of hay, each trip around the field gets shorter. On the first trip around the field, there were 267 bales of hay. On the second trip, there were 253. The number of bales on each succeeding trip decreases arithmetically. The total number of trips is 13. How many bales of hay does the farmer get from the field?



Extra Practice 12.3

Name ______________________

In 1–6, decide whether the sequence is arithmetic, geometric, or neither.

1. $5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \frac{5}{16}, \ldots$
2. $2, \frac{9}{2}, 7, \frac{19}{2}, 12, \frac{29}{2}, \ldots$
3. $2, 5, 10, 13, 26, 29, 58, \ldots$
4. $1, -4, 16, -64, 256, \ldots$
5. $-5, 5, 7, -7, -5, 5, 7, -7, \ldots$
6. $-3, \frac{3}{4}, -\frac{3}{16}, \frac{3}{64}, -\frac{3}{256}, \ldots$

In 7–12, find the common ratio of the geometric sequence. Then, write the next term.

7. $6, 18, 54, 162, 486, \ldots$
8. $2, -8, 32, -128, 512, \ldots$
9. $5, 15, 45, 135, 405, \ldots$
10. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \frac{16}{243}, \ldots$
11. $-2, \frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, -\frac{1}{128}, \ldots$
12. $7, \frac{21}{4}, \frac{63}{16}, \frac{189}{64}, \frac{567}{256}, \ldots$

In 13–18, write the first five terms of the geometric sequence.

13. $a_1 = 6, r = \frac{1}{2}$
14. $a_1 = 1, r = \frac{3}{5}$
15. $a_1 = \frac{2}{9}, r = -6$
16. $a_1 = 7, r = 3$
17. $a_1 = 15, r = \frac{2}{3}$
18. $a_1 = 1, r = \frac{1}{6}$

In 19–24, find the indicated term of the geometric sequence.

19. $a_1 = 4, r = \frac{1}{8}, a_3 =$ [?]
20. $a_1 = 10, r = \frac{1}{5}, a_5 =$ [?]
21. $a_1 = 2, r = 3, a_{12} =$ [?]
22. $a_1 = 4, r = \frac{2}{5}, a_6 =$ [?]
23. $a_1 = 100, r = 4, a_4 =$ [?]
24. $a_1 = \frac{1}{2}, r = \frac{1}{2}, a_{20} =$ [?]

In 25–27, write a formula for the nth term of the geometric sequence.

25. $1, \frac{4}{9}, \frac{16}{81}, \frac{64}{729}, \frac{256}{6561}, \ldots$
26. $100, 5, \frac{1}{4}, \frac{1}{80}, \frac{1}{1600}, \ldots$
27. $-5, 10, -20, 40, -80, \ldots$

In 28–30, write the missing information about the geometric sequence.

28. Common ratio: $\frac{1}{4}$
    First term: 1024
    Sixth term: [?]
29. Third term: 180
    Sixth term: 38,880
    Common ratio: [?]
30. Fifth term: 768
    Seventh term: 12,288
    Formula for a_n: [?]

In 31–33, evaluate the sum.

31. $\sum_{i=1}^{15} 4\left(\frac{1}{2}\right)^{i-1}$
32. $\sum_{i=1}^{10} \frac{1}{2}\left(6^{i-1}\right)$
33. $\sum_{n=0}^{11} 6\left(\frac{1}{2}\right)^{n}$

34. *Salary Plan* Suppose you go to work at a company that pays \$0.01 for the first day, \$0.02 for the second day, \$0.04 for the third day, and so on. (Each day your wage doubles.) What would your total income be if you worked for 10 days? 20 days? 30 days?

AFM

Quiz 1 Review

Name ______________________

Date ____________ Period ________

1. Find the first four terms & the 10th term.

$a_n = \frac{3n}{n-1}$

1st four terms: ______________________ 10th term: ______

2. Find the 1st five terms of the recursively defined sequence.

$a_n = 8 - a_{n-1}$, $a_1 = 3$

1st five terms: ______________________

3. Find the 1st five terms of the recursively defined sequence.

$a_n = 12 + a_{n-1}$, $a_1 = -2$

1st five terms: ______________________

4. How many terms are in the sequence: 5, 8, 11, 14, ..., 155?

n = ________

5. Determine the 98th term for the following arithmetic sequence: 23, 15, 7, -1, -9, ...

a_{98} = ________

6. The 12th term of an arithmetic sequence is 98 and the 3rd term is 71. Find the 33rd term.

a_{33} = ________

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Quiz 1 Practice

Name ______________________

Date ____________ Period ________

1. Find the first four terms & the 50th term.
   $a_n = (-1)^{n-1} \cdot (3n)$

   1st four terms: ______________________ 50th term: ________

2. Find the 1st four terms if:
   $a_n = 2(a_{n-1}) + 4,\ a_1 = -6$

   1st four terms: ______________________

3. Determine the nth term of the sequence 2, 8, 14, 20, ...

   a_n= ______________________

4. How many terms are in the sequence: -5, -8, -11, ..., -47?

   n = ________

5. The 19th term of an arithmetic sequence is 41 and the 27th term is 57. Find the 42nd term.

   a_{42} = ________

6

12.3 Geometric Sequences Date ____________

Geometric sequence – sequence in which each term is ____________________ by a ______________ ______________ that is a ____________________ to get the next te

$a, \; ar, \; ar^2, \; ar^3, \; ar^4, \; ....$

a: ______________ r: ____________________

n^{th} term: $a_n =$ ____________________

Examples:

Find r and the next two terms of the geometric sequence.

1. 4, 2, 1, ... _____, _____

   r = _____

2. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, ...$ _____, _____

   r = _____

3. Find the next 3 terms in the sequence.

   3, -6, 12, -24, ...

4. Find the nth term

   a =3, r = 2, $a_n =$ ____________________

5. Find the 7^{th} term of

   -1, -4, -16, ...

6. $2, -10, \; 50, -250, \; 1250, ...$

   a = _______, r = _______, $a_n =$ ____________________, $a_{10} =$ ____________________

7

7. Find the 1st term given $a_8 = 64$, $r = 2$.

8. Find the 7th term if $a_3 = 96$ & $r = 4$.

9. $a_3 = \frac{63}{4}$, $a_6 = \frac{1701}{32}$, $a_5 = ?$

 Date ____________

In general, S_n stands for the n[th] partial sum (the sum of the first n terms).

For arithmetic sequences, $S_n = \frac{n}{2}(a + a_n)$ or $S_n = \frac{n}{2}(2a + (n-1)d)$

Examples:

1. Find the sum of the 1st 40 terms of: 3, 7, 11, 15, ...

2. Find the sum of the 1st 50 even numbers.

3. An amphitheater has 50 rows with 30 seats in the first row, 32 seats in the second row, 34 seats in the 3rd row, and so on. Find the total number of seats.

4. Find the partial sum of the sequence: 3, 5, 7, 9, ..., 111.

5. How many terms must be added in 5, 7, 9, ... to get a sum of 572?

12.3 Sums of Geometric Sequences

Date ______________

For geometric sequences, $S_n = a \cdot \frac{1-r^n}{1-r}$

Examples:

1. Find the sum of 1[st] 5 terms of: 1, 0.7, 0.49, ...

2. Find the sum of the 1[st] 12 terms of: 7, -14, 28, -56, ...

Sum of an Infinite Geometric Series

If half of a pizza is left and you keep taking ½ of what is left over and over, eventually, the sum of the pizza that has been taken will approach 1.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$ approaches 1. Even though you continue to add more and more terms, the sum will never go beyond 1 because the amount you are adding is less than 1.

In a geometric series, if $|r| < 1$, the sum of an infinite amount of terms is $S_\infty = \frac{a}{1-r}$.

(If if $|r|$ is not less than 1, then there is no infinite sum ... the sum will keep getting bigger and bigger. An infinite sum in that case would not exist.)

Example:

1. Find the sum (if it exists) of: $2+\frac{2}{5}+\frac{2}{25}+\frac{2}{125}+\cdots$

2. Find the sum (if it exists) of: $\frac{3}{4}+3+12+48+\cdots$

AFM Module 8
Partial Sums

Name ____________________

Find the partial sum of each geometric sequence.

1. a = -2, r = -2, n = 8

2. a = 4, r = -3, n = 8

3. a = 3, r = -2, n = 9

4. a =4 , r = $\frac{2}{5}$, n = 6

5. 6 + 12 + 24 + ... + 6144
   *6144 is the 11th term

6. 1 + 3 + 9 + ... + 6561
   *6561 is the 9th term

Find the partial sum of each arithmetic sequence.

7. a = 2, d = 3, n = 7

8. a = 25, d =-8 , n = 8

9. a = -3, d = 4, n = 5

10. a_2=8, a_5=9.5, n = 15

11. 1 + 5 + 9 + ... + 405
    *405 is the 102nd term

12. 1 + 5 + 9 + ... + 413
    *413 is the 104th term

Sigma Notation (Summation Notation):

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

is read "The sum of a_k from k=1 to k=n." [k is called the index.]

It means to add the terms of the sequence starting with the lowest k value up until the n^{th} term.

Examples: For 1 – 2, find each sum.

1. $\sum_{k=1}^{4} k^2$

2. $\sum_{k=3}^{7} (2k-1)$

3. Write using sigma notation: 5 + 11 + 17 + 23 + 29 + ... + 83

AFM Quiz 2 Review

State whether each sequence is arithmetic, geometric, or neither. If it is geometric, find the common ratio, fifth term, and nth term.

1. $5, 10, 20, 40, \ldots$

2. $300, 30, 3, 0.3, \ldots$

3. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

4. $20, 30, 45, \frac{135}{2}, \ldots$

5. $6, -6, 6, -6, \ldots$

6. $-1, \sqrt{5}, -5, 5\sqrt{5}, \ldots$

7. A geometric sequence has a sixth term of 24 and a common ratio of $\frac{2}{3}$. Find the first 3 terms.

$a_1 =$ ________ $a_2 =$ ________ $a_3 =$ ________

8. Which term of the geometric sequence $3, 15, 75, \ldots$ is $46,875$?

n=________

9. Find the partial sum of the geometric sequence in which $a_1 = 4, r = -3, and\ n = 10$.

$a_n =$ ________

10. Find the partial sum of the arithmetic sequence $2 + 9 + 16 + \ldots + 100$.

$S_n =$ ________

11. Find the partial sum of the geometric sequence in which $a = 50, r = -3$ and $n = 10$.

$S_n =$ ________

12. Find the sum of :

a. $\sum_{x=1}^{6} 2^{x-1}$

b. $\sum_{x=3}^{12} (2 + 3x)$

c. $\sum_{x=0}^{6} 5\left(\frac{2}{3}\right)^x$

AFM – Sum Practice
Infinite Geometric Sums & Sigma Notation

Name ____________________

Find the sum of each sequence.

1. $\sum_{k=3}^{8} k$

5. $\sum_{k=1}^{3} (k^2 + 4)$

2. $\sum_{k=1}^{4} -k$

6. $\sum_{k=0}^{4} (k^2 - 4)$

3. $\sum_{k=1}^{5} (5k + 3)$

7. $\sum_{k=1}^{4} (-1)^k 2^k$

4. $\sum_{k=2}^{6} (3k - 7)$

8. $\sum_{k=0}^{3} (k^3 + 2)$

Express each sum using summation notation.

1. 1 + 2 + 3 + ... + 20

2. 4 + 7 + 10 + ... +34

3. 3 + 6 + 12 + ... + 192

AFM – Sum Practice
Infinite Geometric Sums & Sigma Notation

Name ___________________

Decide whether the infinite series has a sum.

1. $10 + 20 + 40 + \ldots$

2. $3 + \frac{12}{5} + \frac{48}{25} + \ldots$

3. $\sum_{k=1}^{\infty} 3(\frac{7}{2})^{k-1}$

4. $\sum_{k=1}^{\infty} -4(\frac{1}{6})^{k-1}$

5. $\sum_{k=1}^{\infty} 5(-\frac{2}{5})^{k-1}$

6. $\sum_{k=1}^{\infty} (\frac{8}{7})^{k-1}$

Find the sum of the series (if it has one).

1. $\sum_{k=1}^{\infty} 2(\frac{2}{3})^{k-1}$

2. $\sum_{k=1}^{\infty} 12(\frac{1}{4})^{k-1}$

3. $\sum_{k=1}^{\infty} 5(\frac{4}{9})^{k-1}$

4. $\sum_{k=1}^{\infty} \frac{1}{8}(8)^{k-1}$

5. $\sum_{k=1}^{\infty} 10(-\frac{1}{2})^{k-1}$

6. $\sum_{k=1}^{\infty} -5(0.1)^{k-1}$

Module 8 Extra Test Review

1. The 14[th] term of an arithmetic sequence is 39.7 and the 3[rd] term is 6.7. Find the common difference and the 32[nd] term for this sequence.

$d =$ ________

$a_{32} =$ ________

2. Find the sum of the arithmetic series $3 + 5 + 7 + \cdots + 51$.

$S_n =$ ________

3. Which term of the arithmetic sequence $-1.8, -1.6, -1.4, \ldots$ is 0.8?

$n =$ ________

4. Find the first four terms and the 15[th] term of the sequence: $a_n = \frac{-1}{3n+4}$

$a_1 =$ ________ $a_2 =$ ________ $a_{15} =$ ________

$a_3 =$ ________ $a_4 =$ ________

5. Write out the sum without using sigma notation:

$$\sum_{j=0}^{4} 2j - 3$$

____

6. Find the sum of the infinite geometric series $3 + \frac{3}{8} + \frac{3}{64} + \frac{3}{512} + \cdots$

$S_\infty =$ ________

7. The third term of the geometric sequence is 2 and the 9th term is $\frac{1}{2048}$. Find the common ratio and the first term of this sequence.

$r =$ ________

$a_1 =$ ________

8. The first term of the geometric sequence is $\frac{1}{2}$ and the second term is 1. Find the sixth term.

$a_6 =$ ________

9. How many terms are in the sequence $0.75, 1.5, 3, \ldots, 3145728$?

$n =$ ________

10. A Kleenex display has 48 rows of Kleenex boxes. There are 7 on the top row, 13 in the second row, 19 in the third row and so on. How many boxes are in row 28? How many total boxes are in the display?

$a_{28} =$ ________

$S_n =$ ________

AFM-Module 8
Review Sheet

Name:________________________
Date:________________ Pd:________

1. Find the first four terms as well as the 1000th term of the sequence $a_n = \frac{1}{4n^2}$

$a_1 =$ __________

$a_2 =$ __________

$a_3 =$ __________

$a_4 =$ __________

$a_{1000} =$ __________

2. Find the first five terms of the recursively defined sequence $a_n = \frac{a_{n-1}}{2}$ where $a_1 = -64$

$a_1 =$ __________

$a_2 =$ __________

$a_3 =$ __________

$a_4 =$ __________

$a_5 =$ __________

3. Find the nth term of the sequence: $3, -9, 27, -81, \ldots$

$a_n =$ __________

4. Find the nth term of the sequence: $a, a-3d, a-6d, a-9d, \ldots$

$a_n =$ __________

5. Find the sum of:

$$\sum_{n=1}^{6} \frac{3}{2n}$$

$S_n =$ __________

6. Find the sum of:

$$\sum_{k=3}^{7} k(k+4)$$

$S_n =$ __________

7. Write the sum $3+6+9+\cdots+54$ in sigma notation.

8. Determine the common difference, the fifth term, the nth term, and the 100th term for the arithmetic sequence $12.7, 9.3, 5.9, 2.5, \ldots$

$d =$ __________

$a_5 =$ __________

$a_n =$ __________

$a_{100} =$ __________

9. The 11th term of an arithmetic sequence is 80 and the fifth term is 56. Find the common difference for this sequence.

$d =$ __________

10. Find the sum (S_n) of the arithmetic series that satisfies the conditions $a = 75, d = -2, n = 8$.

$S_n =$ __________

11. Find the sum of the arithmetic series $-7 + \left(\frac{-11}{2}\right) + (-4) + \left(\frac{-5}{2}\right) + (-1) + \cdots + 29$

$S_n =$ __________

12. The 7th term of an arithmetic sequence is -18 and the 17th term is -68. Find the 24th term.

$a_{24} =$ __________

13. Which term of the arithmetic sequence $\frac{1}{2}, 2, \frac{7}{2}, \ldots$ *is* 38?

____

14. A large outdoor amphitheater has 40 rows of seats with 30 seats in the first row, 35 seats in the second, 40 in the third and so on.

a. How many seats are in row 35? __________

b. How many seats are there in the theater? __________

15. Determine the common ratio, the fifth term, and the nth term for the geometric sequence $7, \frac{14}{5}, \frac{28}{25}, \frac{56}{125}, \ldots$

$r =$ ______________

$a_5 =$ ______________

$a_n =$ ______________

16. The first four terms of a sequence are $\frac{-5}{2}, \frac{-1}{3}, \frac{-2}{9}, \frac{-4}{27} \ldots$ Determine whether the terms can be the terms of an arithmetic sequence, a geometric sequence, or neither. Find the next term if the sequence is arithmetic/geometric.

Type: ______________

Next term = ______________

17. The first term of a geometric sequence is 27 and the second term is 3. Find the fifth term.

$a_5 =$ ______________

18. The common ratio in a geometric sequence is $\frac{3}{4}$ and the fifth term is 2. Find the first three terms.

$a_1 =$ ______________

$a_2 =$ ______________

$a_3 =$ ______________

19. Which term of the geometric sequence $3, 12, 48, \ldots$ is 786,432?

____

20. Find the sum of the infinite geometric series $\frac{1}{4} + \frac{3}{20} + \frac{9}{100} + \frac{27}{500} \ldots$

$S_n =$ ______________

21. Which of the following geometric series are convergent?

a. $-4 + 4 - 4 + 4 - \cdots$

b. $6 + 4 + \frac{8}{3} + \frac{16}{9} + \ldots$

c. $19 + 16 + 13 + 10 + \cdots$

d. $18 - 9 + 4.5 - 2.25 + \cdots$

The following properties of sums are natural consequences of properties of the real numbers.

PROPERTIES OF SUMS

Let $a_1, a_2, a_3, a_4, \ldots$ and $b_1, b_2, b_3, b_4, \ldots$ be sequences. Then for every positive integer n and any real number c, the following properties hold.

1. $\displaystyle\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$
2. $\displaystyle\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$
3. $\displaystyle\sum_{k=1}^{n} ca_k = c\left(\sum_{k=1}^{n} a_k\right)$

Proof To prove Property 1, we write out the left side of the equation to get

$$\sum_{k=1}^{n} (a_k + b_k) = (a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3) + \cdots + (a_n + b_n)$$

Because addition is commutative and associative, we can rearrange the terms on the right side to read

$$\sum_{k=1}^{n} (a_k + b_k) = (a_1 + a_2 + a_3 + \cdots + a_n) + (b_1 + b_2 + b_3 + \cdots + b_n)$$

Rewriting the right side using sigma notation gives Property 1. Property 2 is proved in a similar manner. To prove Property 3, we use the Distributive Property:

$$\sum_{k=1}^{n} ca_k = ca_1 + ca_2 + ca_3 + \cdots + ca_n$$

$$= c(a_1 + a_2 + a_3 + \cdots + a_n) = c\left(\sum_{k=1}^{n} a_k\right)$$

□

12.1 EXERCISES

1–10 ■ Find the first four terms and the 1000th term of the sequence.

1. $a_n = n + 1$
2. $a_n = 2n + 3$
3. $a_n = \dfrac{1}{n+1}$
4. $a_n = n^2 + 1$
5. $a_n = \dfrac{(-1)^n}{n^2}$
6. $a_n = \dfrac{1}{n^2}$
7. $a_n = 1 + (-1)^n$
8. $a_n = (-1)^{n+1}\dfrac{n}{n+1}$
9. $a_n = n^n$
10. $a_n = 3$

11–16 ■ Find the first five terms of the given recursively defined sequence.

11. $a_n = 2(a_{n-1} - 2)$ and $a_1 = 3$

12. $a_n = \frac{a_{n-1}}{2}$ and $a_1 = -8$

13. $a_n = 2a_{n-1} + 1$ and $a_1 = 1$

14. $a_n = \frac{1}{1 + a_{n-1}}$ and $a_1 = 1$

15. $a_n = a_{n-1} + a_{n-2}$ and $a_1 = 1, a_2 = 2$

16. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ and $a_1 = a_2 = a_3 = 1$

17–24 ■ Find the nth term of a sequence whose first several terms are given.

17. $2, 4, 8, 16, \ldots$

18. $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots$

19. $1, 4, 7, 10, \ldots$

20. $5, -25, 125, -625, \ldots$

21. $1, \frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \ldots$

22. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$

23. $0, 2, 0, 2, 0, 2, \ldots$

24. $1, \frac{1}{2}, 3, \frac{1}{4}, 5, \frac{1}{6}, \ldots$

25–28 ■ Find the first six partial sums $S_1, S_2, S_3, S_4, S_5, S_6$ of the sequence.

25. $1, 3, 5, 7, \ldots$

26. $1^2, 2^2, 3^2, 4^2, \ldots$

27. $\frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}, \frac{1}{3^4}, \ldots$

28. $-1, 1, -1, 1, \ldots$

29–32 ■ Find the first four partial sums and the nth partial sum of the sequence a_n.

29. $a_n = \frac{2}{3^n}$

30. $a_n = \frac{1}{n+1} - \frac{1}{n+2}$

31. $a_n = \sqrt{n} - \sqrt{n+1}$

32. $a_n = \log\left(\frac{n}{n+1}\right)$ [*Hint:* Use a property of logarithms to write the nth term as a difference.]

33–40 ■ Find the sum.

33. $\sum_{k=1}^{4} k$

34. $\sum_{k=1}^{4} k^2$

35. $\sum_{k=1}^{3} \frac{1}{k}$

36. $\sum_{j=1}^{100} (-1)^j$

37. $\sum_{i=1}^{8} [1 + (-1)^i]$

38. $\sum_{i=4}^{12} 10$

39. $\sum_{k=1}^{5} 2^{k-1}$

40. $\sum_{i=1}^{3} i2^i$

41–46 ■ Write the sum without using sigma notation.

41. $\sum_{k=1}^{5} \sqrt{k}$

42. $\sum_{i=0}^{4} \frac{2i-1}{2i+1}$

43. $\sum_{k=0}^{6} \sqrt{k+4}$

44. $\sum_{k=6}^{9} k(k+3)$

45. $\sum_{k=3}^{100} x^k$

46. $\sum_{j=1}^{n} (-1)^{j+1} x^j$

47–54 ■ Write the sum using sigma notation.

47. $1 + 2 + 3 + 4 + \cdots + 100$

48. $2 + 4 + 6 + \cdots + 20$

49. $1^2 + 2^2 + 3^2 + \cdots + 10^2$

50. $\frac{1}{2 \ln 2} - \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} - \frac{1}{5 \ln 5} + \cdots + \frac{1}{100 \ln 100}$

51. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{999 \cdot 1000}$

52. $\frac{\sqrt{1}}{1^2} + \frac{\sqrt{2}}{2^2} + \frac{\sqrt{3}}{3^2} + \cdots + \frac{\sqrt{n}}{n^2}$

53. $1 + x + x^2 + x^3 + \cdots + x^{100}$

54. $1 - 2x + 3x^2 - 4x^3 + 5x^4 + \cdots - 100x^{99}$

55. Find a formula for the nth term of the sequence

$$\sqrt{2},\ \sqrt{2\sqrt{2}},\ \sqrt{2\sqrt{2\sqrt{2}}},\ \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}},\ \ldots$$

[*Hint:* Write each term as a power of 2.]

56. Fibonacci posed the following problem: Suppose that rabbits live forever and that every month each pair produces a new pair that becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the nth month? Show that the answer is F_n, where F_n is the nth term of the Fibonacci sequence.

DISCOVERY • DISCUSSION

57. Different Sequences That Start the Same

(a) Show that the first four terms of the sequence $a_n = n^2$ are

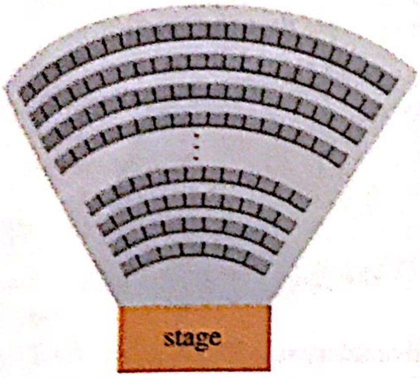
$$1, 4, 9, 16, \ldots$$

(b) Show that the first four terms of the sequence $a_n = n^2 + (n-1)(n-2)(n-3)(n-4)$ are also

$$1, 4, 9, 16, \ldots$$

Substituting in Formula 2 for the partial sum of an arithmetic sequence, we get

$$S_{50} = 50\left(\frac{a + a_{50}}{2}\right) = 50\left(\frac{1 + 99}{2}\right) = 50 \cdot 50 = 2500$$ ■



EXAMPLE 6 ■ Finding the Seating Capacity of an Amphitheater

An amphitheater has 50 rows of seats with 30 seats in the first row, 32 in the second, 34 in the third, and so on. Find the total number of seats.

SOLUTION

The numbers of seats in the rows form an arithmetic sequence with $a = 30$ and $d = 2$. Since there are 50 rows, the total number of seats is the sum

$$S_{50} = \tfrac{50}{2}[2(30) + 49(2)] \qquad \text{Formula 1}$$

$$= 3950$$

Thus, the amphitheater has 3950 seats. ■

EXAMPLE 7 ■ Finding the Number of Terms in a Partial Sum

How many terms of the arithmetic sequences 5, 7, 9, . . . must be added to get 572?

SOLUTION

We are asked to find n when $S_n = 572$. Substituting $a = 5$, $d = 2$, and $S_n = 572$ in Formula 1 for the partial sum of an arithmetic sequence, we get

$$572 = \frac{n}{2}[2 \cdot 5 + (n - 1)2] \qquad \text{Formula 1}$$

$$572 = 5n + n(n - 1)$$

$$0 = n^2 + 4n - 572$$

$$0 = (n - 22)(n + 26)$$

This gives $n = 22$ or $n = -26$. But since n is the *number* of terms in this partial sum, we must have $n = 22$. ■

12.2 EXERCISES

1–6 ■ Determine whether the sequence is arithmetic. If it is arithmetic, find the common difference.

1. 5, 8, 11, 14, . . .

2. 3, 6, 9, 13, . . .

3. 2, 4, 8, 16, . . .

4. 2, 4, 6, 8, . . .

5. $3, \frac{3}{2}, 0, -\frac{3}{2}, \ldots$

6. ln 2, ln 4, ln 8, ln 16, . . .

7–16 ■ Determine the common difference, the fifth term, the nth term, and the 100th term of the arithmetic sequence.

7. 2, 5, 8, 11, . . .

8. 1, 5, 9, 13, . . .

9. 4, 9, 14, 19, . . .

10. 11, 8, 5, 2, . . .

11. −12, −8, −4, 0, . . .

12. $\frac{7}{6}, \frac{5}{3}, \frac{13}{6}, \frac{8}{3}, \ldots$

13. 25, 26.5, 28, 29.5, . . .

14. 15, 12.3, 9.6, 6.9, . . .

15. $2, 2 + s, 2 + 2s, 2 + 3s, \ldots$

16. $-t, -t + 3, -t + 6, -t + 9, \ldots$

17. The tenth term of an arithmetic sequence is $\frac{55}{2}$, and the second term is $\frac{7}{2}$. Find the first term.

18. The 12th term of an arithmetic sequence is 32, and the fifth term is 18. Find the 20th term.

19. The 100th term of an arithmetic sequence is 98, and the common difference is 2. Find the first three terms.

20. The 20th term of an arithmetic sequence is 101, and the common difference is 3. Find a formula for the nth term.

21. Which term of the arithmetic sequence 1, 4, 7, . . . is 88?

22. The first term of an arithmetic sequence is 1, and the common difference is 4. Is 11,937 a term of this sequence? If so, which term is it?

23–28 ■ Find the partial sum S_n of the arithmetic sequence that satisfies the given conditions.

23. $a = 1, d = 2, n = 10$

24. $a = 3, d = 2, n = 12$

25. $a = 4, d = 2, n = 20$

26. $a = 100, d = -5, n = 8$

27. $a_1 = 55, d = 12, n = 10$

28. $a_2 = 8, a_5 = 9.5, n = 15$

29–34 ■ A partial sum of an arithmetic sequence is given. Find the sum.

29. $1 + 5 + 9 + \cdots + 401$

30. $-3 + \left(-\frac{3}{2}\right) + 0 + \frac{3}{2} + 3 + \cdots + 30$

31. $0.7 + 2.7 + 4.7 + \cdots + 56.7$

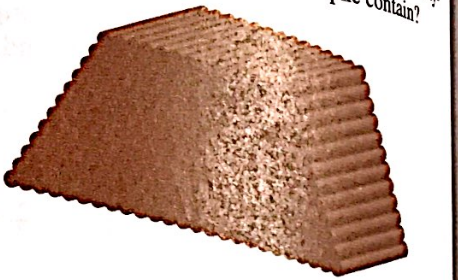
32. $-10 - 9.9 - 9.8 - \cdots - 0.1$

33. $\sum_{k=0}^{10} (3 + 0.25k)$

34. $\sum_{n=0}^{20} (1 - 2n)$

35. The purchase value of an office computer is \$12,500. Its annual depreciation is \$1875. Find the value of the computer after 6 years.

36. Telephone poles are stored in a pile with 25 poles in the first layer, 24 in the second, and so on. If there are 12 layers, how many telephone poles does the pile contain?



37. A man gets a job with a salary of \$30,000 a year. He is promised a \$2300 raise each subsequent year. Find his total earnings for a 10-year period.

38. A drive-in theater has spaces for 20 cars in the first parking row, 22 in the second, 24 in the third, and so on. If there are 21 rows in the theater, find the number of cars that can be parked.

39. An architect designs a theater with 15 seats in the first row, 18 in the second, 21 in the third, and so on. If the theater is to have a seating capacity of 870, how many rows must the architect use in his design?

40. An arithmetic sequence has first term $a = 5$ and common difference $d = 2$. How many terms of this sequence must be added to get 2700?

41. When an object is allowed to fall freely near the surface of the earth, the gravitational pull is such that the object falls 16 ft in the first second, 48 ft in the next second, 80 ft in the next second, and so on.

(a) Find the total distance a ball falls in 6 s.

(b) Find a formula for the total distance a ball falls in n seconds.

42. In the well-known song "The Twelve Days of Christmas," a person gives his sweetheart k gifts on the kth day for each of the 12 days of Christmas. The person also repeats each gift identically on each subsequent day. Thus, on the 12th day the sweetheart receives a gift for the first day, 2 gifts for the second, 3 gifts for the third, and so on. Show that the number of gifts received on the 12th day is a partial sum of an arithmetic sequence. Find this sum.

43. Show that a right triangle whose sides are in arithmetic progression is similar to a 3–4–5 triangle.

11. $144, -12, 1, -\frac{1}{12}, \ldots$

12. $-8, -2, -\frac{1}{2}, -\frac{1}{8}, \ldots$

13. $3, 3^{5/3}, 3^{7/3}, 27, \ldots$

14. $t, \dfrac{t^2}{2}, \dfrac{t^3}{4}, \dfrac{t^4}{8}, \ldots$

15. $1, s^{2/7}, s^{4/7}, s^{6/7}, \ldots$

16. $5, 5^{c+1}, 5^{2c+1}, 5^{3c+1}, \ldots$

17. The first term of a geometric sequence is 8, and the second term is 4. Find the fifth term.

18. The first term of a geometric sequence is 3, and the third term is $\frac{4}{3}$. Find the fifth term.

19. The common ratio in a geometric sequence is $\frac{2}{5}$, and the fourth term is $\frac{5}{2}$. Find the third term.

20. The common ratio in a geometric sequence is $\frac{3}{2}$, and the fifth term is 1. Find the first three terms.

21. Which term of the geometric sequence 2, 6, 18, . . . is 118,098?

22. The second and the fifth terms of a geometric sequence are 10 and 1250, respectively. Is 31,250 a term of this sequence? If so, which term is it?

23–26 ■ Find the partial sum S_n of the geometric sequence that satisfies the given conditions.

23. $a = 5, \quad r = 2, \quad n = 6$

24. $a = \frac{2}{3}, \quad r = \frac{1}{3}, \quad n = 4$

25. $a_3 = 28, \quad a_6 = 224, \quad n = 6$

26. $a_2 = 0.12, \quad a_5 = 0.00096, \quad n = 4$

27–30 ■ Find the sum.

27. $1 + 3 + 9 + \cdots + 2187$

28. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{512}$

29. $\displaystyle\sum_{k=0}^{10} 3\left(\tfrac{1}{2}\right)^k$

30. $\displaystyle\sum_{j=0}^{5} 7\left(\tfrac{3}{2}\right)^j$

31. A ball is dropped from a height of 80 ft. The elasticity of this ball is such that it rebounds three-fourths of the distance it has fallen. How high does the ball rebound on the fifth bounce? Find a formula for how high the ball rebounds on the *n*th bounce.

32. A culture initially has 5000 bacteria, and its size increases by 8% every hour. How many bacteria are present at the end of 5 hours? Find a formula for the number of bacteria present after *n* hours.

33. A truck radiator holds 5 gal and is filled with water. A gallon of water is removed from the radiator and replaced with a gallon of antifreeze; then, a gallon of the mixture is removed from the radiator and again replaced by a gallon of antifreeze. This process is repeated indefinitely. How much water remains in the tank after this process is repeated 3 times? 5 times? *n* times?

34. A very patient woman wishes to become a billionaire. She decides to follow a simple scheme: She puts aside 1 cent the first day, 2 cents the second day, 4 cents the third day, and so on, doubling the number of cents each day. How much money will she have at the end of 30 days? How many days will it take this woman to realize her wish?

35. A ball is dropped from a height of 9 ft. The elasticity of ball is such that it always bounces up one-third the dista it has fallen.

(a) Find the total distance the ball has traveled at the instant it hits the ground the fifth time.

(b) Find a formula for the total distance the ball has tra eled at the instant it hits the ground the *n*th time.

36. The following is a well-known children's rhyme:

As I was going to St. Ives
I met a man with seven wives;
Every wife had seven sacks;
Every sack had seven cats;
Every cat had seven kits;
Kits, cats, sacks, and wives,
How many were going to St. Ives?

Assuming that the entire group is actually going to St. show that the answer to the question in the rhyme is a tial sum of a geometric sequence, and find the sum.

37–44 ■ Find the sum of the infinite geometric series.

37. $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \cdots$

38. $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \cdots$

39. $1 - \dfrac{1}{3} + \dfrac{1}{9} - \dfrac{1}{27} + \cdots$

40. $\dfrac{2}{5} + \dfrac{4}{25} + \dfrac{8}{125} + \cdots$

41. $\dfrac{1}{3^6} + \dfrac{1}{3^8} + \dfrac{1}{3^{10}} + \dfrac{1}{3^{12}} + \cdots$

42. $3 - \dfrac{3}{2} + \dfrac{3}{4} - \dfrac{3}{8} + \cdots$