Binomial Theorem

$(x+y)^{12}$

$(x+y)^2 = (x+y)(x+y)$

$x^2+xy+xy+y^2$

$x^2+2xy+y^2$

$(x+y)^3 = (x+y)(x+y)(x+y)$

$(x+y)(x^2+2xy+y^2)$

$x^3+2x^2y+xy^2+x^2y+2xy^2+y^3$

$x^3+3x^2y+3xy^2+y^3$

use only if needed

$(x+y)^4 = (x+y)(x+y)(x+y)(x+y)$

$(x+y)(x^3+3x^2y+3xy^2+y^3)$

$x^4+3x^3y+3x^2y^2+xy^3+x^3y+3x^2y^2+3xy^3+y^4$

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

Note: 1^{st} position: exponents decrease

2^{nd} position: exponents increase

$\boxed{\text{use Combinations}}$

1) $(x+y)^4$ $\quad n=4$ $\quad$ r starts at $0 \rightarrow r =$ exponent of 2^{nd} #

1^{st}	${}_4C_0$	x^4y^0	$= 1x^4 \cdot 1$
2^{nd}	${}_4C_1$	x^3y^1	$= 4x^3y$
3^{rd}	${}_4C_2$	x^2y^2	$= 6x^2y^2$
4^{th}	${}_4C_3$	x^1y^3	$= 4xy^3$
5^{th}	${}_4C_4$	x^0y^4	$= 1 \cdot 1y^4$

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

2.) $(2x+5)^4$ $n=4$

1^{st} ${}_4C_0$ $(2x)^4(5)^0 = 1 \cdot 16x^4 \cdot 1$

2^{nd} ${}_4C_1$ $(2x)^3(5)^1 = 4 \cdot 8x^3 \cdot 5$

3^{rd} ${}_4C_2$ $(2x)^2(5)^2 = 6 \cdot 4x^2 \cdot 25$

4^{th} ${}_4C_3$ $(2x)^1(5)^3 = 4 \cdot 2x \cdot 125$

5^{th} ${}_4C_4$ $(2x)^0(5)^4 = 1 \cdot 1 \cdot 625$

$16x^4 + 160x^3 + 600x^2 + 1000x + 625$

3.) $(x-y)^5$

1^{st} ${}_5C_0$ $(x^5)^5(-y)^0 = 1 \cdot x^5 \cdot 1$

2^{nd} ${}_5C_1$ $x^4(-y)^1 = 5 \cdot x^4 \cdot -y$

3^{rd} ${}_5C_2$ $x^3(-y)^2 = 10 \cdot x^3 \cdot y^2$

4^{th} ${}_5C_3$ $x^2(-y)^3 = 10 \cdot x^2 \cdot -y^3$

5^{th} ${}_5C_4$ $x^1(-y)^4 = 5 \cdot x \cdot y^4$

6^{th} ${}_5C_5$ $x^0(-y)^5 = 1 \cdot 1 \cdot -y^5$

$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

$(3x-5y)^5$

	C's	x's	y's	term
${}_5C_0\,(3x)^5\,(-5y)^0$	$=1$	$243x^5$	1	$243x^5$
${}_5C_1\,(3x)^4\,(-5y)^1$	$=5$	$81x^4$	$-5y$	$-2025x^4y$
${}_5C_2\,(3x)^3\,(-5y)^2$	$=10$	$27x^3$	$25y^2$	$6750x^3y^2$
${}_5C_3\,(3x)^2\,(-5y)^3$	$=10$	$9x^2$	$-125y^3$	$-11250x^2y^3$
${}_5C_4\,(3x)^1\,(-5y)^4$	$=5$	$3x$	$625y^4$	$9375xy^4$
${}_5C_5\,(3x)^0\,(-5y)^5$	$=1$	1	$-3125y^5$	$-3125y^5$

$$243x^5-2025x^4y+6750x^3y^2-11250x^2y^3+9375xy^4-3125y^5$$

4. Given $(2x+5)^{12}$ find the 4^{th} term

n = ~~#~~ exponent r = term # -1

(count on hand, start w/ 0 + 1 finger)

$_{12}C_3$ $4-1=3$

4^{th} term

Alt. $x^{12}\cdot 5^0$

$x^{11}\cdot 5^1$

$x^{10}\cdot 5^2$

$x^9\, 5^3$

$_{12}C_3\,(2x)^{9}(5)^{3}$ ← r goes there

1^{st} position + 2^{nd} must add to n!

$220 \cdot 512x^9 \cdot 125$

$14080000x^9$

5. Given $(x-3y)^{10}$, find the 8^{th} term

$_{10}C_7\,(x)^3(-3y)^7$

$120 \cdot x^3 \cdot -2187y^7$

$-262440x^3y^7$

$a^{10}\, b^0$

b^1

b^2

b^3

b^4

b^5

b^6

b^7

b^8