Section 6.2 - The Natural Exponential Function

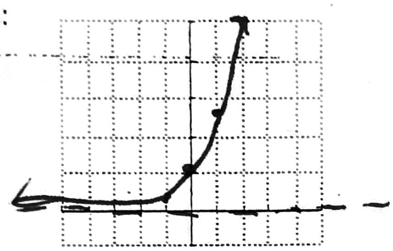
Name: ______________________

Date: __________ Period: __________

Definition: A function of base e

$e \approx 2.7182818$

Graph:



x	y
0	1
1	2.7
−1	$\frac{1}{2.7}$

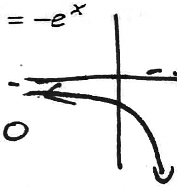
State the domain, range, and asymptote for the following graphs.

1. $y = -e^{x}$

D: ℝ

R: $y<0$

A: $y=0$



2. $y = 1 + e^{x}$

D: ℝ

R: $y>1$

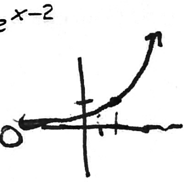
A: $y=1$

3. $y = e^{x-2}$

D: ℝ

R: $y>0$

A: $y=0$

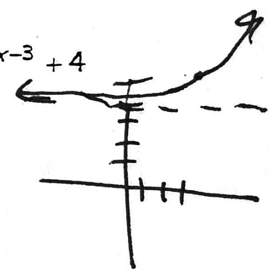


4. $y = e^{x-3} + 4$

D: ℝ

R: $y>4$

A: $y=4$



Compound Interest

Formula:

Total $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$

P = principle

r = rate (decimal)

n = # of times compounded

t = time (years)

Annually - 1

Semiannually - 2

Triannually - 3

Quarterly - 4

Monthly - 12

Weekly - 52

daily - 365

Example: A sum of $1000 is invested at an interest rate of 12% per year. Find the amounts in the accour after 5 years if interest is compounded annually, semiannually, quarterly, monthly, and daily.

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Annually: $1000\left(1+\frac{.12}{1}\right)^{1\cdot 5} = 1762.34$

Semiannually: $1000\left(1+\frac{.12}{2}\right)^{2\cdot 5} = 1790.85$

Quarterly: $1000\left(1+\frac{.12}{4}\right)^{4\cdot}$ 1806

Monthly: $1000\left(1+\frac{.12}{12}\right)^{12\cdot 5} = 1816.70$

Daily: $1000\left(1+\frac{.12}{365}\right)^{365\cdot 5} = 1821.94$

Continuously Compounded Interest

Formula: $A(t) = Pe^{rt}$

Example: Find the amount after 5 years if $1000 is invested at an interest rate of 12% per year, compounde continuously.

$P = 1000$ $\quad 1000e^{.12 \cdot 5} = 1822.12$

$r = .12$

$t = 5$

Exponential Growth (Population)

Living organism... people, bacteria etc

Formula: $N(t) = N_0 e^{rt}$

$n(t)$ = total Pop
n_0 = Initial Population
r = Rate (decimal)
t = time

Example #1: The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and he finds that the relative rate of growth is 40% per hour.

a) Find a function that models the number of bacteria after t hours. $n = 500e^{(.40t)}$

b) What is the estimated count after 10 hours?

$N = 500e^{.40 \cdot 10} = 27299.08$

Example #2: Under ideal conditions, a certain type of bacteria has a relative growth rate of 220% per hour. A number of these bacteria are introduced accidentally into a food product. Two hours after contamination, a bacterium count shows that there are about 40,000 bacteria in the food.

a) Find the initial number of bacteria introduced into the food.

$N = N_0 e^{rt}$

$40000 = N_0 e^{(2.2 \cdot 2)}$

$40000 = N_0 e^{4.4}$

$\frac{40000}{e^{4.4}} = N_0$

$N_0 = 491.09$

b) Estimate the number of bacteria in the food 3 hours after contamination.

$N = 491 e^{(2.2 \cdot 3)}$

36093.74