Theoretical Probability – what should happen

Empirical Probability – what really happens

The more trials conducted, the closer the empirical approaches the theoretical number.

Expected Value – The average amount a person could expect to win for every game played.

The game is <u>fair</u> if the expected value is 0.

Simple version $P(E) \cdot \text{Amt Win} = \text{Expected Value}$

* Lookout for loss and cost to play

Complex

$P(\text{win}) \cdot \text{Amt} - P(\text{loss}) \cdot \text{Amt} - \text{cost to play}$.

Expected Value

The projected winnings for a given game based on the probability of winning and the amount you would win.

Example 1: A spinner has 8 slots you can land on. 2 green slots are marked winners. You have to pay $2 to play. If you land on a green winner you will win $5. What is the expected value?

	Win	Lose
Outcome	Green	Not Green
Probability	2/8	6/8
Payoff	$5	0

$$P(\text{Green}) \cdot \text{Amt} - P(\text{not green}) \cdot \text{Amt} - \text{Cost to play}$$
$$\frac{2}{8} \cdot 5 - \frac{6}{8} \cdot 0 - 2 = -.75$$

Example 2: If the sum of two rolled die is 8 or more, you win $2, if not, you lose $1. Is this game fair?

$$P(8+) \quad P(\text{not } 8+)$$
$$\frac{15}{36} \cdot 2 - \frac{21}{36} \cdot 1 = .25$$

No. You expect to win more than you lose.

Example 3: Two coins are tossed. If both land heads up, then player A wins $4 from player B. If exactly 1 coin lands heads up, then B wins $1 from A. If both land tails up then B wins $2 from A. Is this game fair?

Use Player A's perspective

	H	T
H	HH	HT
T	TH	TT

$$\frac{1}{4} \cdot 4 - \frac{2}{4} \cdot 1 - \frac{1}{4} \cdot 2 = 0$$

Because the "ev" is 0, the game is fair.

You expect to win as much as you lose.

AFM

Name _____

Unit 3 ~ Expected Value Practice

1. You draw one card from a standard deck of playing cards. If you pick a heart, you will win $10. If you pick a face card, which is not a heart, you win $8. If you pick any other card, you lose $6. Do you want to play? Explain.

$\frac{13}{52} \cdot 10 + \frac{9}{52} \cdot 8 + \frac{30}{52} \cdot -6 = .42$ EV is positive, Yes! play the game.

2. The world famous gambler from Philadelphia, Señor Ross, proposes the following game of chance. You roll a fair die. If you roll a 1, then Señor Ross pays you $25. If you roll a 2, Señor Ross pays you $5. If you roll a 3, you win nothing. If you roll a 4 or a 5, you must pay Señor Ross $10, and if you roll a 6, you must pay Señor Ross $15. Is Señor Ross loco for proposing such a game? Explain.

$\frac{1}{6} \cdot 25 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 0 + \frac{2}{6} \cdot -10 - \frac{1}{6} \cdot 15 = -.83$

3. You pay $10 to play the following game of chance. There is a bag containing 12 balls, five are red, three are green and the rest are yellow. You are to draw one ball from the bag. You will win $14 if you draw a red ball and you will win $12 is you draw a yellow ball. How much do you expect to win or loss if you play this game 100 times?

4. Detective Stevens figures that he has a one in nine chance of recovering stolen property. His out of-pocket expenses for the investigation are $9,000. If he is paid his fee only if he recovers the stolen property, what should he charge clients in order to breakeven?

$\frac{1}{9}x - 9000 = 0$

5 At Tucson Raceway Park, your horse, Soon-to-be-Glue, has a probability of $\frac{1}{20}$ of coming in first place, a probability of $\frac{1}{10}$ of coming in second place, and a probability of $\frac{1}{4}$ of coming in third place. First place pays $4,500 to the winner, second place $3,500 and third place $1,500. Is it worthwhile to enter the race if it costs $1,000?

6. Your company plans to invest in a particular project. There is a 35% chance that you will lose $30,000, a 40% chance that you will break even, and a 25% chance that you will make $55,000. Based solely on this information, what should you do?

$-.35 \cdot 30000 + .40 \cdot 0 + .25 \cdot 55000$

7. A company is considering the manufacturing of a new and improved mousetrap. The company estimates the probability that the new mousetrap is successful is $\frac{3}{4}$. If it is successful it would generate profits of $120,000. The development costs for the mousetrap are $98,000. Should the company proceed with plans for the new mousetrap? Why or why not?

8. A grab bag contains 12 packages worth 80 cents apiece, 15 packages worth 40 cents apiece and 25 packages worth 30 cents apiece. Is it worthwhile to pay 50 cents for the privilege of picking one of the packages at random?