Date_____

Systems of Non-linear Equations

Use sep sheet for space

Solve the following systems of equations. When necessary, round your answers to the nearest hundredth.

1. $2x^2 + y^2 = 18$
   $xy = 4$

$y = \frac{4}{x}$

$2x^2 + \left(\frac{4}{x}\right)^2 = 18$

$2x^2 + \frac{16}{x^2} = 18$

$2x^4 + 16 = 18x^2$

$2x^4 - 18x^2 + 16 = 0$

$x^4 - 9x^2 + 8 = 0$

$(x^2 - 8)(x^2 - 1) = 0$

$x^2 = 8 \quad x^2 = 1$

$x = \pm\sqrt{8} \quad x = \pm 1$

$(1, 4)$

$(-1, -4)$

$(2.83, 1.41)$

$(-2.83, -1.41)$

2. $y = 2x + 1$
   $2x^2 + y^2 = 1$

$2x^2 + (2x+1)^2 = 1$

$2x^2 + 4x^2 + 4x + 1 = 1$

$6x^2 + 4x = 0$

$2x(3x + 2) = 0$

$2x = 0 \quad 3x + 2 = 0$

$x = 0 \quad 3x = -2$

$x = -\frac{2}{3}$

$y = 0 + 1 = 1$

$y = 2\left(-\frac{2}{3}\right) + 1$

$-\frac{1}{3}$

$(0, 1)$

$\left(-\frac{2}{3}, -\frac{1}{3}\right)$

3. $x^2 - 4y^2 = 16$
   $2y - x = 2$

$x = 2y - 2$

$(2y - 2)^2 - 4y^2 = 16$

$4y^2 - 8y + 4 - 4y^2 = 16$

$-8y + 4 = 16$

$-8y = 12$

$y = -3/2$ $\quad x = -5$

$x = 2(-3/2) - 2$

$-6 - 2$

-8

$\left(-\frac{4}{3}, -\frac{8}{3}\right)$

4. $4x^2 + 3y^2 = 4$
   $2x^2 - 6y^2 = -3$

$4x^2 + 3y^2 = 4$

$4x^2 - 12y^2 = -6$

$15y^2 = 10$

$y^2 = 2/3$

$4x^2 + 3\left(\frac{2}{3}\right) = 4$

$4x^2 + 2 = 4$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{1/2}$

$y = \pm\sqrt{2/3}$

$\left(\sqrt{1/2}, \sqrt{2/3}\right)$

$\left(-\sqrt{1/2}, \sqrt{2/3}\right)$

$\left(\sqrt{1/2}, -\sqrt{2/3}\right)$

$\left(-\sqrt{1/2}, -\sqrt{2/3}\right)$

5. $2x^2 + 2xy = 10$
   $3x^2 - xy = 2$

$t + a = xy$

$2x^2 + 2a = 10$

$3x^2 - a = 2$

$x^2 + a = 5$

$3x^2 - a = 2$

$4x^2 = 7$

$x^2 = \frac{7}{4}$

$x = \pm\frac{\sqrt{7}}{2}$

$3\left(\frac{\sqrt{7}}{2}\right)^2 - a = 2$

$\frac{21}{4} - a = 2$

$-a = -13/4$

$a = 13/4$

$\frac{13}{4} = \frac{\sqrt{7}}{2}y \quad \frac{13}{4} = \frac{-\sqrt{7}}{2}y$

$\frac{13}{2\sqrt{7}} = y \quad \frac{-13}{2\sqrt{7}} = y$

$\left(\frac{\sqrt{7}}{2}, \frac{13}{2\sqrt{7}}\right), \left(-\frac{\sqrt{7}}{2}, \frac{-13}{2\sqrt{7}}\right)$

6. $y = x^2 - 4$
   $y = 6x - 13$

$x^2 - 4 = 6x - 13$

$x^2 - 6x + 9 = 0$

$(x - 3)(x - 3) = 0$

$x = 3$

$y = 3^2 - 4 = 5$

$(3, 5)$

7. $2x^2 + y^2 = 2$
   $x^2 - 2y^2 + 8 = 0$

$x^2 + y^2 = 2$

$x^2 - 2y^2 = -8$

$2x^2 + y^2 = 2$

$2x^2 - 4y^2 = -16$

$5y^2 = 18$

$y^2 = \frac{18}{5}$

$y = \pm\sqrt{\frac{18}{5}}$

$2x^2 + \frac{18}{5} = 2$

$2x^2 = -\frac{8}{5}$

$x^2 = -\frac{4}{5}$

$x = \pm\frac{2i}{\sqrt{5}}$

$\left(\frac{2i}{\sqrt{5}}, \sqrt{18/5}\right)$

$\left(\frac{-2i}{\sqrt{5}}, \sqrt{18/5}\right)$

$\left(\frac{2i}{\sqrt{5}}, -\sqrt{18/5}\right)$

$\left(\frac{-2i}{\sqrt{5}}, -\sqrt{18/5}\right)$

8. $xy = 4$
   $x^2 + y^2 = 8$

$y = \frac{4}{x}$

$x^2 + \left(\frac{4}{x}\right)^2 = 8$

$x^2 + \frac{16}{x^2} = 8$

$x^4 + 16 = 8x^2$

$x^4 - 8x^2 + 16 = 0$

$(x^2 - 4)(x^2 - 4) = 0$

$x = \pm 2$

$y = \frac{4}{2} = 2$ $\quad y = \frac{4}{-2} = -2$

$(2, 2)$ $\quad (-2, -2)$

9. The difference of two numbers is 2 and the sum of their squares is 10. Find the numbers.

$x - y = 2$

$x^2 + y^2 = 10$

$x = 2 + y$

$(2 + y)^2 + y^2 = 10$

$y^2 + 4y + 4 + y^2 = 10$

$2y^2 + 4y - 6 = 0$

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y = -3 \quad y = 1$

$x = 3 \quad y = 1$

$x = -1 \quad y = -3$

10. The sum of two numbers is 7 and the difference of their squares is 21. Find the numbers.

$x + y = 7$

$x^2 - y^2 = 21$

$x = 7 - y$

$(7 - y)^2 - y^2 = 21$

$49 - 14y + y^2 - y^2 = 21$

$49 - 14y = 21$

$-14y = -28$

$y = 2$

$x + 2 = 7$

$x = 5$

$(5, 2)$

11. The product of two numbers is 10 and the difference of their squares is 21. Find the numbers.

$xy = 10$

$x^2 - y^2 = 21$

$y = \frac{10}{x}$

$x^2 - \left(\frac{10}{x}\right)^2 = 21$

$x^2 - \frac{100}{x^2} = 21$

$x^4 - 100 = 21x^2$

$x^4 - 21x^2 - 100 = 0$

$(x^2 - 25)(x^2 + 4) = 0$

$x^2 = 25 \quad x^2 = -4$

$x = \pm 5 \quad x = \pm 2i$

$5y = 10 \quad -5y = 10$

$y = 2 \quad y = -2$

$(5, 2) \quad (-5, -2)$