Name _____

1. How many different outcomes can a race with 6 runners have, assuming there is no tie?

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 720$

$6!$

2. A coin is flipped 5 times, and the resulting sequence of heads and tails is recorded. How many different sequences are possible?

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 32$

3. An employee ID number system consists of one letter followed by 2 digits. How many different ID numbers are possible with this system?

$\underline{26}\ \underline{10}\ \underline{10} = 2600$

4. An automobile dealer offers 4 models. Each model comes in a choice of 5 colors, 6 types of stereo equipment, with or without air conditioning, and with or without a sunroof. How many different ways can a customer order an auto from this dealer?

$\underset{\text{mod}}{\underline{5}}\ \underset{\text{col}}{\underline{4}}\ \underset{\text{ste}}{\underline{6}}\ \underset{\text{A/C}}{\underline{2}}\ \underset{\text{sun}}{\underline{2}} = 480$

5. How many ways can 7 people be seated in a row of 7 chairs, if Jack insists on sitting in the first chair?

$\underline{1}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 720$

6. Find the number of distinguishable permutations of the given letters

ADHLLLPP

$\frac{8!}{3!\ 2!} = 3360$

7. How many possibilities are there in selecting a committee of 4 members from a club of 35 members?

$35C4 = 52360$

8. A violinist has practiced 15 pieces. How many different combinations are possible if he chooses 2 of these pieces for a recital?

$15C2 = 105$

9. The president of a large company periodically selects one employee to receive a special bonus. He claims that this employee is chosen randomly from among the 56 employees, of which 29 are women and 27 are men. What is the probability that no woman is chosen?

$$\frac{{}_{27}C_1 \cdot {}_{29}C_0}{{}_{56}C_1} = \frac{27-1}{56}$$

10. A jar contains 14 red marbles numbered 1 to 14, and 29 blue marbles numbered 1 to 29. A marble is drawn at random from the jar. Find the probability that the marble is blue or even-numbered.

$$\frac{29}{43} + \frac{21}{43} - \frac{14}{43} = \frac{36}{43}$$

11. A slot machine has three wheels, and each wheel has 11 positions: the digits 0 to 9 and a picture of a watermelon. When a quarter is placed in the machine and the handle is pulled, the three wheels spin independently and come to rest. When three watermelons show, the payout is $450 otherwise, nothing is paid out. What is the expectedvalue of this game? $\frac{1}{11} \cdot \frac{1}{11} \cdot \frac{1}{11} \cdot 450 - .25 = .09$

12. How many three letter "words" (strings of letters) can be formed using the letters *A G M R* if repetition of letters is allowed?

$4 \cdot 4 \cdot 4 = 64$

13. An all-star baseball team has a roster of 6 pitchers and 2 catchers. How many pitcher-catcher pairs can the manager select from this roster?

$6 \cdot 2 = 12$

14. How many different combinations of people can be chosen as president, vice president, and secretary of a class of 28 students?

$${}_{28}P_3 = 28 \cdot 27 \cdot 26 = 19656$$

15. How many different combinations are possible in choosing a president, vice president, and secretary from a class of 16 females and 28 males if the president must be a female and the vice president a male?

$16 \cdot 28 \cdot 42 = 18816$

16. Some three-digit numbers are formed using the digits 5, 6, 7, and 9, with repetition of digits allowed. How many such numbers can be formed if the value of each number can go no higher than 900?

$3 \cdot 4 \cdot 4 = 48$

17. Evaluate the expression:

$$P(9, 2) = {}_9P_2 = 72$$

18. A pianist plans to play 10 pieces at a recital. How many ways can she arrange the program by playing these pieces in different orders?

$${}_{10}P_{10} = 10! = 3628800$$

19. How many ways can 3 blue marbles and 5 red marbles be arranged in a row?

bbb rrrrr

3 5

$$\frac{8!}{3!5!} = 56$$

20. A man bought 2 vanilla ice cream cones, 3 chocolate cones, 4 strawberry cones, and 5 butterscotch cones for his 14 children. How many different ways can he distribute the cones among his children?

VV CCC SSSS bbbbb

2 3 4 5

$$\frac{14!}{2!3!4!5!} = 2522520$$

21. Evaluate the expression:

$${}_7C_2 = C(7, 2) = 21$$

22. How many different 7-card hands can be selected from a deck of 52 cards?

Order doesn't matter

$${}_{52}C_7 = 133,784,560$$

23. An experiment consists of tossing a coin and rolling a die. Find the probability of getting tails and a number greater than 4. H 1 2 3 4 5 6 / H 1 H 2 H 3 H 4 H 5 H 6 H / T 1 T 2 T 3 T 4 T 5 T 6 T $\frac{1}{2} \cdot \frac{2}{6} = \frac{2}{12} = \frac{1}{6}$

24. A ball is drawn randomly from a jar that contains 2 red balls, 8 white balls, and 4 yellow balls. What is the probability that the ball selected is not white? 14 total, 6 not white $\frac{6}{14} = \frac{3}{7}$

25. A roulette wheel has 24 slots; two slots are numbered 0 and 00, and the remaining slots are numbered from 1 to 22. Find the probability that the ball lands in an odd–numbered slot.

26. A die is rolled twice. What is the probability of showing 5 on the first roll and an odd number on the second roll? $\frac{1}{6} \cdot \frac{3}{6} = \frac{3}{36} = \frac{1}{12}$

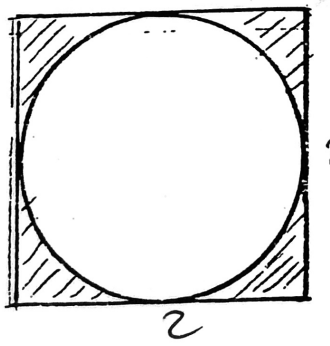
27. Jane wins \$16 if a die roll shows a six, and she loses \$1 otherwise. Find the expected value (or expectation) of the game. $\frac{1}{6} \cdot 16 - \frac{5}{6} \cdot 1 = 1.83$

28. A die is rolled. Tom wins \$9 if the die shows an even number, otherwise, he pays \$9. Find the expected value (or expectation) of the game. $\frac{3}{6} \cdot 9 - \frac{3}{6} \cdot 9 = 0$

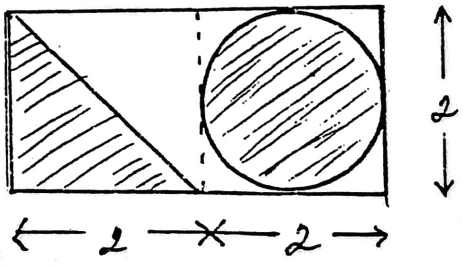
29. A card is drawn from a deck. You win \$124 if the card is an ace, \$42 if it is a face card, and \$15 if it is the 8 of clubs. Find the expected value (or expectation) of the game. $\frac{4}{52} \cdot 124 + 42 \cdot \frac{12}{52} + \frac{1}{52} \cdot 15 = 19.52$

30. A bag contains eight white balls and two black balls. John picks two balls at random from the bag, and he wins \$4 if he does not pick a black ball. Find the expected value (or expectation) of the game.

31. A box contains 100 envelopes. Ten envelopes contain \$10 each, ten contain \$5 each, two are "unlucky", and the rest are empty. A player draws an envelope from the box and keeps whatever is in it. If a person draws an unlucky envelope, however, he must pay \$100. What is the expectation of a person playing this game? $\frac{10}{100} \cdot 10 + \frac{10}{100} \cdot 5 - \frac{2}{100} \cdot 100 = -.50$



Area
square = 4
Circle = π



Rect: 8
circle: π
$\Delta = \frac{1}{2} \cdot 2 \cdot 2 = 2$

32. In the above figure at left, if a point is chosen at random, what is the probability it will be:

a. inside the circle $\frac{\pi}{4} = .7854$ b. in the shaded region $\frac{4-\pi}{4} = .2146$

33. In the above figure at right, if a point is chosen at random, what is the probability it will be:

a. in the white area $\frac{8-\pi-2}{8} = .3573$ b. in the shaded area $\frac{\pi+2}{8} = .6427$

~~34.~~ A box contains 3 red balls and 2 green balls. Two balls are randomly chosen without replacement. If just one ball is green, you win \$1, otherwise you lose \$1. What is your expected gain or loss?

$\frac{1}{5} \cdot 1 - \frac{4}{5} \cdot \frac{1}{4} = 0$

35. On a multiple choice test a student is given five possible answers for each question. The student receives 1 point for a correct answer and loses ¼ point for an incorrect answer. If the student has no idea of the correct answer for a particular question and merely guesses, what is the student's expected gain or loss on the question?

36. Expand: $(p+q)^6$

$_6C_0\ p^6\ q^0 \quad 1\ p^6$
$_6C_1\ p^5\ q^1 \quad 6\ p^5 q$
$_6C_2\ p^4\ q^2 \quad 15\ p^4 q^2$
$_6C_3\ p^3\ q^3 \quad 20\ p^3 q^3$
$_6C_4\ p^2\ q^4 \quad 15\ p^2 q^4$
$_6C_5\ p^1\ q^5 \quad 6\ p\ q^5$
$_6C_6\ p^0\ q^6 \quad 1\ q^6$

$p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 +$

37. Expand: $(x-y)^5$

$_5C_0\ x^5(-y)^0 = x^5$
$_5C_1\ x^4(-y) = -5x^4y$
$_5C_2\ x^3(-y)^2 = 10x^3y^2$
$_5C_3\ x^2(-y)^3 = -10x^2y^3$
$_5C_4\ x^1(-y)^4 = 5xy^4$
$_5C_5\ x^0(-y)^5 = -y^5$

$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

38. What is the third term of $(x+3)^{12}$?

$_{12}C_2\ x^{10}\ 3^2$

39. What is the fourth term of $(x+2)^5$?

$_5C_2\ x^2(2)^3$

$10 \cdot x^2 \cdot 8$

$80x^2$