Module 5 Review Sheet: Piecewise Functions Name _____

#1-5 Identify the independent and dependent variables.

1) The cost, in dollars, to fill up my gas tank, in gallons, with unleaded fuel

Independent: # of gallons Dependent: Cost

2) The height of a humans as a function of age

Independent: age Dependent: height

3) The price of a ribeye steak at Harris Teeter

Independent: weight Dependent: Cost or price

4) The amount of money in a paycheck of an hourly employee at Harris Teeter

Independent: hours worked Dependent: amt of money

5) The price of a cheese pizza from Domino's ?

Independent: size Dependent: price

6) Evaluate

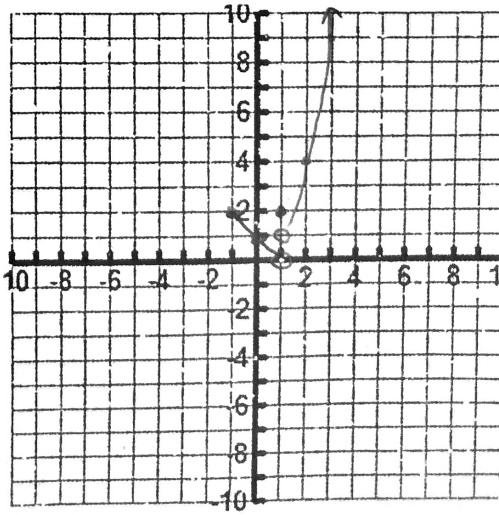
$$f(x) = \begin{cases} x^2 - 5 & -9 < x \le -4 \\ \frac{1}{2}x + 7 & -4 < x < 8 \\ -3 & x \ge 8 \end{cases}$$

a) $f(5) =$ $\frac{25}{2}$ b) $f(-7) =$ 44 c) $f(12) =$ -3

d) $f(-15) =$ undef e) $f(8) =$ -3 f) $f(4) =$ 9

#7 – 10 Sketch the graph of the following functions and answer the related questions.

7) $$f(x) = \begin{cases} -x + 1 & -1 \le x < 1 \\ 2 & x = 1 \\ x^2 & x > 1 \end{cases}$$



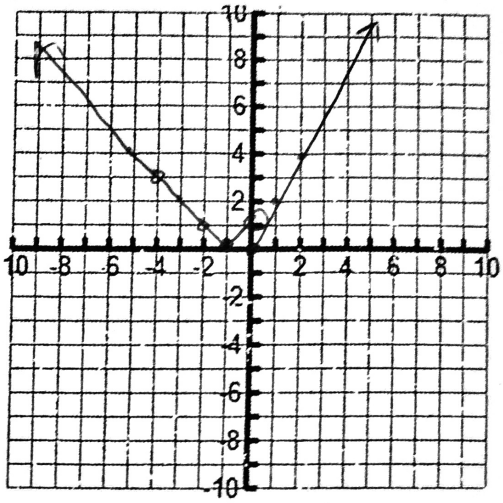
Domain: $x \ge -1$

Range: $y | y > 0$

Interval(s) of increasing: $x | x > 1$

Interval(s) of decreasing: $x | -1 < x < 1$

8) $f(x) = \begin{cases} |x+1| & x < 0 \\ 2x & x \geq 0 \end{cases}$



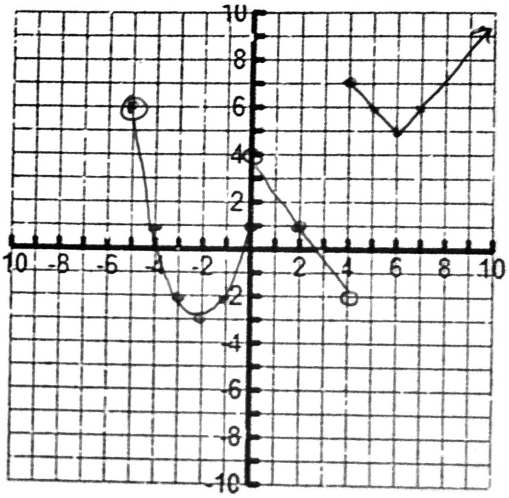
Domain: $x \mid x = \mathbb{R}$

Range: $y \mid y \geq 0$

Interval(s) of increasing: $x \mid x > -1$

Interval(s) of decreasing: $x \mid x < -1$

9) $f(x) = \begin{cases} (x+2)^2 - 3 & -5 < x \leq 0 \\ -\frac{3}{2}x + 4 & 0 < x < 4 \\ |x-6| + 5 & x \geq 4 \end{cases}$



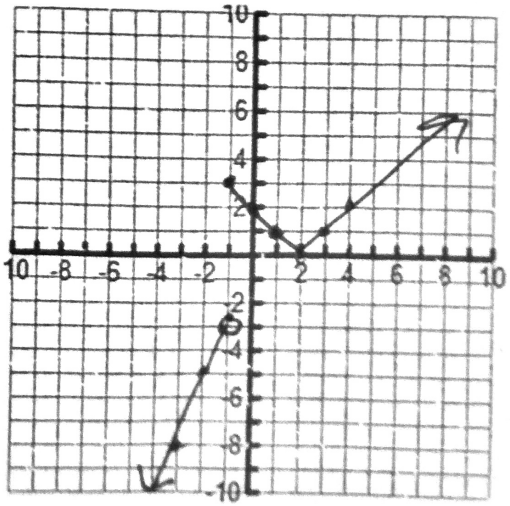
Domain: $x \mid x > -5$

Range: $y \mid y \geq -3$

Interval(s) of increasing: $x \mid -2 < x < 0 \cup x > 6$

Interval(s) of decreasing: $x \mid -5 < x < -2 \cup 0 < x < 4 \cup 4 < x < 6$

10) $f(x) = \begin{cases} 3x & x < -1 \\ |x-2| & x \geq -1 \end{cases}$



Domain: $x \mid x = \mathbb{R}$

Range: $y \mid y < -3 \cup y \geq 0$

Interval(s) of increasing: $x \mid x < -1 \cup x > 2$

Interval(s) of decreasing: $x \mid -1 < x < 2$

Solve each system of equations. Round to the nearest hundredth. Show work on your own paper.

11) $2x^2 + y^2 = 18$
$xy = 4$

12) $x^2 + y^2 = 4$
$y - x^2 = -9$

13) $x^2 - y^2 = 21$
$x + y = 7$

14) $x^2 + y^2 = 29$
$xy = 10$

Write a system of equations for each of the following situations. Then solve the system algebraically or graphically. Do not just guess check!

15) The sum of two numbers is 8 and the sum of their squares is 34. Find the numbers.

System of Equations: $x+y=8$ $x^2+y^2=34$ Solution: 3, 5

16) The sum of two numbers is 10 and the difference of their squares is 50. Find the numbers.

System of Equations: $x+y=10$ $x^2-y^2=50$ Solution: 7.5, 2.5

17) The product of two numbers is 24 and the sum of their squares is 73. Find the numbers.

System of Equations: $xy=24$ $x^2+y^2=73$ Solution: 3, 8

Answer the following appropriately for each application.

18) A rental car company charges a flat fee of \$40 for the first 50 miles drive and an additional \$0.25 per mile for each mile driven in excess of 50 miles.

a) Express the cost, C, as a function of the miles driven, m.
$\$40, 0 < m < 50$ $40 + .25(m-50), m \geq 50$

b) Sketch and label the graph of $C(m)$.

c) Determine the cost of driving 200 miles.
$40 + .25(200-50) = \$77.50$

d) Determine how many miles one could drive for \$100.
$40 + .25(x-50) = 100$
$.25(x-50) = 60$
$x - 50 = 240$
$x = 290$
290 miles

10. $2x^2+y^2=18$

$xy=4$

$y=\frac{4}{x}$

$2x^2+\left(\frac{4}{x}\right)^2=18$

$2x^2+\frac{16}{x^2}=18$

$2x^4+16=18x^2$

$2x^4-18x^2+16=0$

$x^4-9x^2+8=0$

$(x^2-8)(x^2-1)=0$

$x^2=8 \quad x^2=1$

$x=\pm\sqrt{8} \quad x=\pm 1$

$y=\frac{4}{\sqrt{8}} \quad y=\frac{4}{1}$

$y=\frac{4}{-\sqrt{8}} \quad y=\frac{4}{-1}$

$(1,4)$
$(-1,-4)$
$(2.82,1.41)$
$(-2.82,-1.41)$

11. $x^2-y^2=21$

$x+y=7$

$y=7-x$

$x^2-(7-x)^2=21$

$x^2-(49-14x+x^2)=21$

$x^2-49+14x-x^2=21$

$14x-49=21$

$14x=70$

$x=5$

$(5,2)$

12. $x^2+y^2=4$

$y-x^2=-9$

$y+9=x^2$

$y+9+y^2=4$

$y^2+y+5=0$

$\frac{-1\pm\sqrt{1^2-4\cdot1\cdot5}}{2\cdot1}$

no real solution

13. $x^2+y^2=29$

$xy=10$

$y=\frac{10}{x}$

$x^2+\left(\frac{10}{x}\right)^2=29$

$x^2+\frac{100}{x^2}=29$

$x^4+100=29x^2$

$x^4-29x^2+100=0$

$(x^2-4)(x^2-25)=0$

$x^2=4 \quad x^2=25$

$x=\pm 2 \quad x=\pm 5$

$(2,5) \quad (5,2)$
$(-2,-5) \quad (-5,-2)$

14. $x+y=8$

$x^2+y^2=34$

$x^2+(8-x)^2=34$

$x^2+64-16x+x^2=34$

$2x^2-16x+30=0$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$(3,5) \quad (5,3)$

15. $x+y=10$

$x^2-y^2=50$

$x^2-(10-x)^2=50$

$x^2-(100-20x+x^2)=50$

$x^2-100+20x-x^2=50$

$20x-150=0$

$20x=150$

$x=\frac{15}{2}$

$(7.5, 2.5)$

16. $xy=24$

$x^2+y^2=73$

$y=\frac{24}{x}$

$x^2+\left(\frac{24}{x}\right)^2=73$

$x^2+\frac{576}{x^2}=73$

$x^4+576=73x^2$

$x^4-73x^2+576=0$

$(x^2-9)(x^2-64)=0$

$x^2=9 \quad x^2=64$

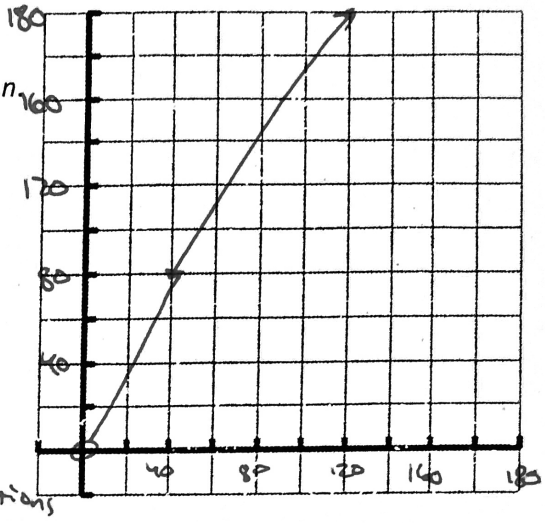
$x=\pm 3 \quad x=\pm 8$

19) Beautiful Wedding charges $2.00 each for printing the first 40 invitations and $1.25 each for each invitation in excess of 40.

a) Express the cost, C, as a function of the number printed, n.

$2x, \ 0 < x \leq 40 \qquad 80 + 1.25(x-40), \ x > 40$

b) Sketch and label the graph of $C(n)$.



c) Determine the cost of printing 275 invitations.

$80 + 1.25(275 - 40) = \$373.75$

d) How many invitations can be ordered for a cost of $300?

$80 + 1.25(x - 40) = 300$

$1.25x - 50 = 220$

$1.25x = 270$

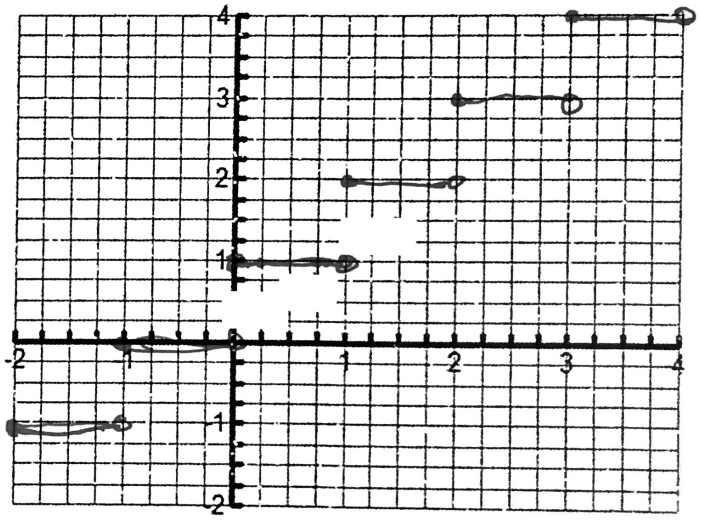
$x = 216$ invitations

Sketch the graph of each function and state the domain and range.

20) $f(x) = [x] + 1$

Domain: $x \mid x = \mathbb{R}$

Range: $y \mid y =$ All integers



21) $f(x) = [2x]$

Domain: $x \mid x = \mathbb{R}$

Range: All integers